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# **ELECTROKINETIC HYDROPHONES**

by

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**TECHNICAL REPORT No. 4**

**TO THE**

**OFFICE OF NAVAL RESEARCH**

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**THE BETA CORPORATION**

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## ELECTROKINETIC HYDROPHONES

### 1. INTRODUCTION

This last technical report is confined to the exploratory and development work carried out under Contract Nonr-617(00) on electrokinetic hydrophones. For reasons of classification specific applications are not discussed. The theory and properties of electrokinetic transducers are discussed in Technical Reports 2 and 3 and in Reference 11.

The results of the work which has been carried out indicate that the most promising field for electrokinetic hydrophones is in low and very low frequency applications where flat response is required and the use of sub-surface preamplifiers is objectionable. Their resistive output impedance and high mechanical input impedance make possible their use at great depths without sub-surface preamplifiers. Their long term stability and ease of calibration may make them useful as standards in frequency ranges from as low as 0.1 cps to 1000 cps. The factors which govern or limit the performance of electrokinetic hydrophones are relatively simple but must be fully understood before a sound decision can be made as to whether to employ an electrokinetic type of transducer in a proposed hydrophone application. These factors are discussed in this report.

A number of electrokinetic hydrophone arrangements were considered and several types were built and tested. The numerical and graphical data on frequency response along with other data on the hydrophones included in this report were obtained by the U.S.N. Underwater Sound Reference Laboratory at Orlando, Florida, at the request of the Acoustics Branch of the Office of Naval Research.

Performance relations involving power sensitivity, impedance, equivalent noise pressure, depth, equalizer volume, low frequency cut-off, etc. are developed and presented as alignment charts in Appendix A. Other notes regarding self-noise and the use of transformers are also included in the Appendices.

## 2. SUMMARY AND CONCLUSIONS

### (a.) Performance Table of Electrokinetic Hydrophones Developed on Project NR 385 407

A table of average characteristics of the hydrophones which were developed is given below. Where additional testing is required to fully establish the correctness of an entry it is marked with an asterisk. The units are the same as those used and defined in Appendix A.

	<u>LF-1</u>	<u>LF-2</u>	<u>S-2</u>
Voltage Sensitivity (DB)	-104	-100	-82.5
Impedance (Reference-Grid)	17K	450K	450K
Power Sensitivity (DB)	-102	-112	-94
Noise pressure-One Cycle (DB)	-52.5	-42.5	-60
Cable Impedance	17K	500	500
Max. Usable Depth (Ft.)	100	20,000*	50
Low Frequency Cut-off (c.p.s.)	0.1	5-10*	20
Upper useful limit (c.p.s.)	1000	1000	1000

Line transformers are available commercially with response flat to 0.1 c.p.s. which will match the LF-1 Hydrophone to a 500 ohm line. It is, therefore, practicable to use any of the above hydrophones with cables several miles in length without sub-surface or cable preamplifiers.

### (b.) Stability

It has been fully established by our own tests, by the U.S.N. Underwater Sound Reference Laboratory tests described in Section 4 and References 2, 3, 4, 5, and 6, and by commercial verifications that electrokinetic transducers or hydrophones constructed as shown in this report hold their calibration for long or indefinite periods of time. (There has been no known instance of a transducer employing the "Type 4" construction losing its calibration.)

### (c.) Accuracy

Transducer calibrations at low frequencies within the mid-band range can readily be carried out to better than 2% or 0.25 DB using a pistonphone. This procedure gives as great or greater accuracy than is usually obtained in commercially available standard hydrophones at low and very low frequencies,

i. e. from 0.1 cps to 1000 cps.

(d.) Economic Factors

No experience has been obtained thus far in the quantity production of electrokinetic hydrophones. To answer questions which may arise it is well to point out that no expensive or critical materials are involved nor are there any close machining tolerances. All critical assembly operations on a production design could be carried out by semi-skilled labor or by automatic machines. The electrokinetic hydrophone, therefore, could be adapted to economical quantity production techniques.

(e.) Future Designs

Many varieties of future designs are possible. It should be possible to predict their performance in advance by considering the factors charted in Appendix A. The hydrophones thus far developed are relatively small and somewhat low on power sensitivity for some applications particularly at frequencies above 100 c.p.s. The sea noise decreases with frequency and the thermal noise as a result becomes comparatively more troublesome at higher frequencies.

One obvious way to raise the power sensitivity and lower the thermal noise pressure is to simply design a larger hydrophone with a larger compressible equalizer volume and multiple parallel or series connected transducing elements. This would appear to be a perfectly practical solution for hydrophones which are to be buried under the sea for long periods and which must operate over long cables without preamplifiers.

It is possible that solid-liquid combinations will be found which will possess increased transducing efficiency. With present combinations efficiency is slightly over 1% with occasional discs giving up to 2%. The term efficiency here refers to the ratio of the mechanical energy converted to electrical energy to the total energy absorbed and dissipated. It is somewhat misleading inasmuch as, by comparison, an electromagnetic moving coil device with no mechanical damping would be 100% efficient. Yet to function in a hydrophone at comparable depths and with flat response over the same frequency ranges mechanical damping would be required to such a degree that the overall efficiency would become very small, particularly at frequencies below, say, 10 cps and at great depths.

It is unlikely that the transducing efficiency will be increased more than a few fold, because of the theoretical relation between efficiency and the double layer thickness to pore radius

ratio. From energy considerations it can be shown that for the same hydrophones described in this report a power sensitivity improvement of 38.6 DB is the maximum theoretically possible for an ideal electrokinetic element or any other kind, other factors being equal. This figure assumes 100% conversion of energy which is most unlikely or impossible in an electrokinetic device.

(f.) Conclusions

It is anticipated that electrokinetic hydrophones will fill a need wherever low or very low frequency pressure response is a requirement, where sub-surface preamplifiers are undesirable, or where low frequency measurements are to be made at great depths or over long periods. Where greater power sensitivity is required it may be obtained by the use of multiple or cylindrical elements, and larger or more compliant equalizers. The thermal noise for electrokinetic transducers at low frequencies, particularly below 10 cps, is substantially lower than that for piezoelectric or ferroelectric transducers of roughly the same dimensions unless extremely high shunt resistive components of impedance are maintained across the latter.

3. MEASUREMENTS WITH A SEALED TRANSDUCER

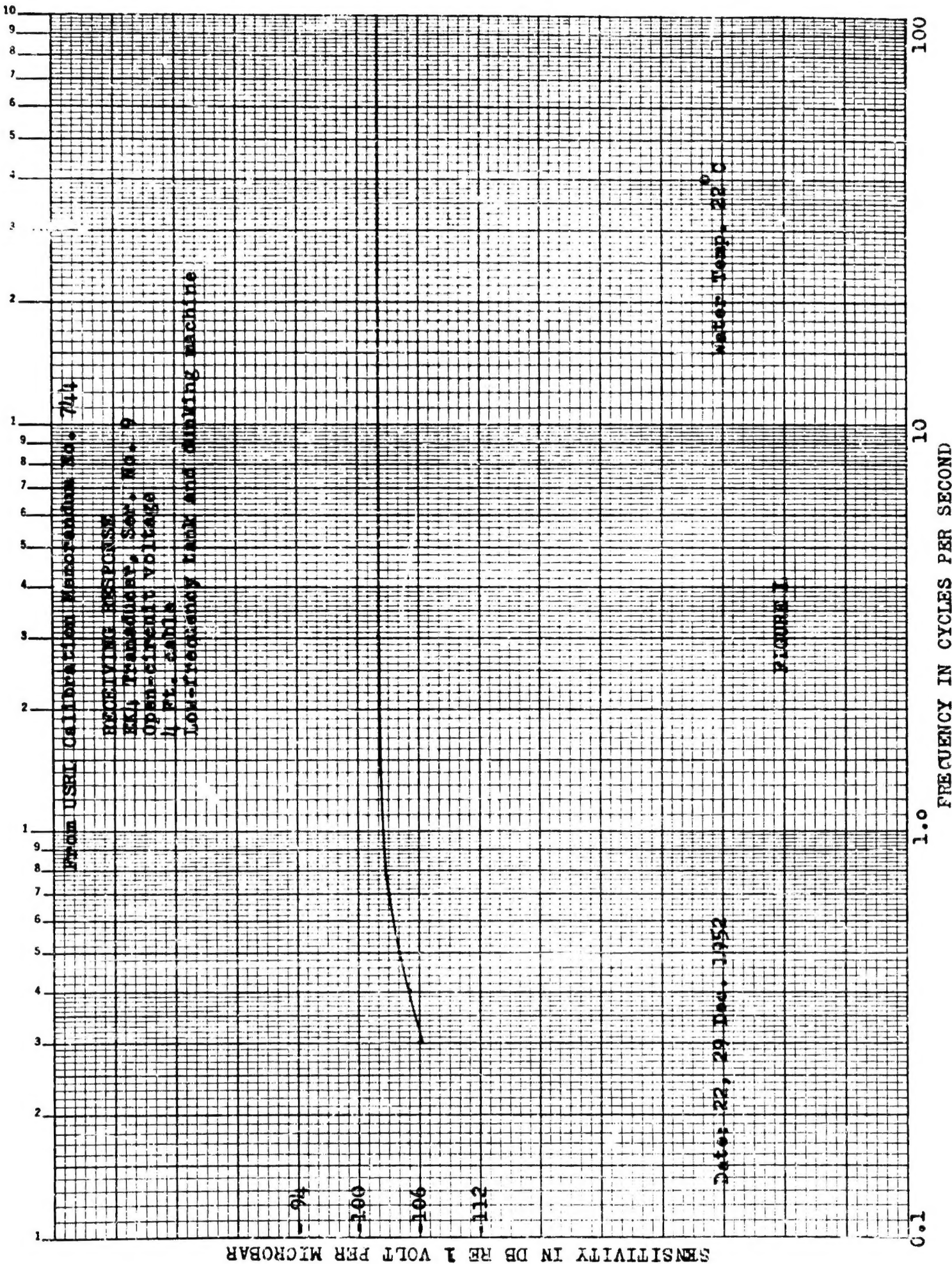
A Type 4 transducer, Serial No. 9, essentially as shown in Figure 3-2 of Technical Report No. 2 was tested by the Underwater Sound Reference Laboratory. A portion of their results as reported in Reference 1 are given in this section.

The transducer employed a 3/4" O.D. by .080" Grade #03 porous porcelain disc in an acetonitrile filled cell. The disc was partially masked with wax improving the low frequency response and raising the impedance by roughly 50%. Brass aluminum overlay diaphragms were employed. A wax and tape seal was employed at the rear joints of the case to last for the duration of the tests. Tests were made with both 20' and 4' cables but only the latter are included here as the longer cable merely attenuated the higher frequencies in a predictable fashion.

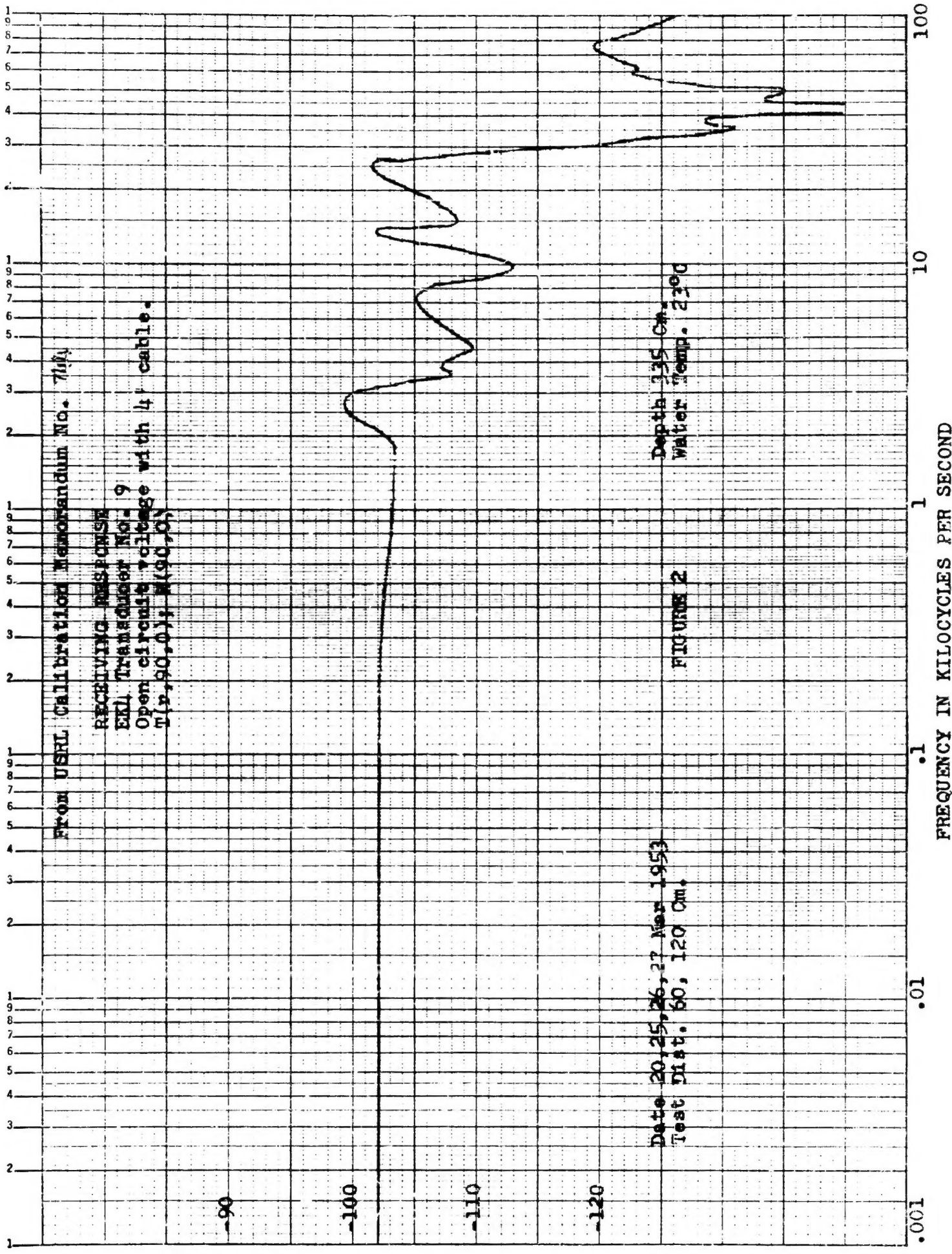
Figures 1 and 2 show the transducers response from 0.3 to 100 c.p.s. and from 1 c.p.s. to 100 KC. (The irregular response from 2KC is peculiar to its underwater characteristics whereas the sudden drop at 30KC appears to be unique to the design in any media.)

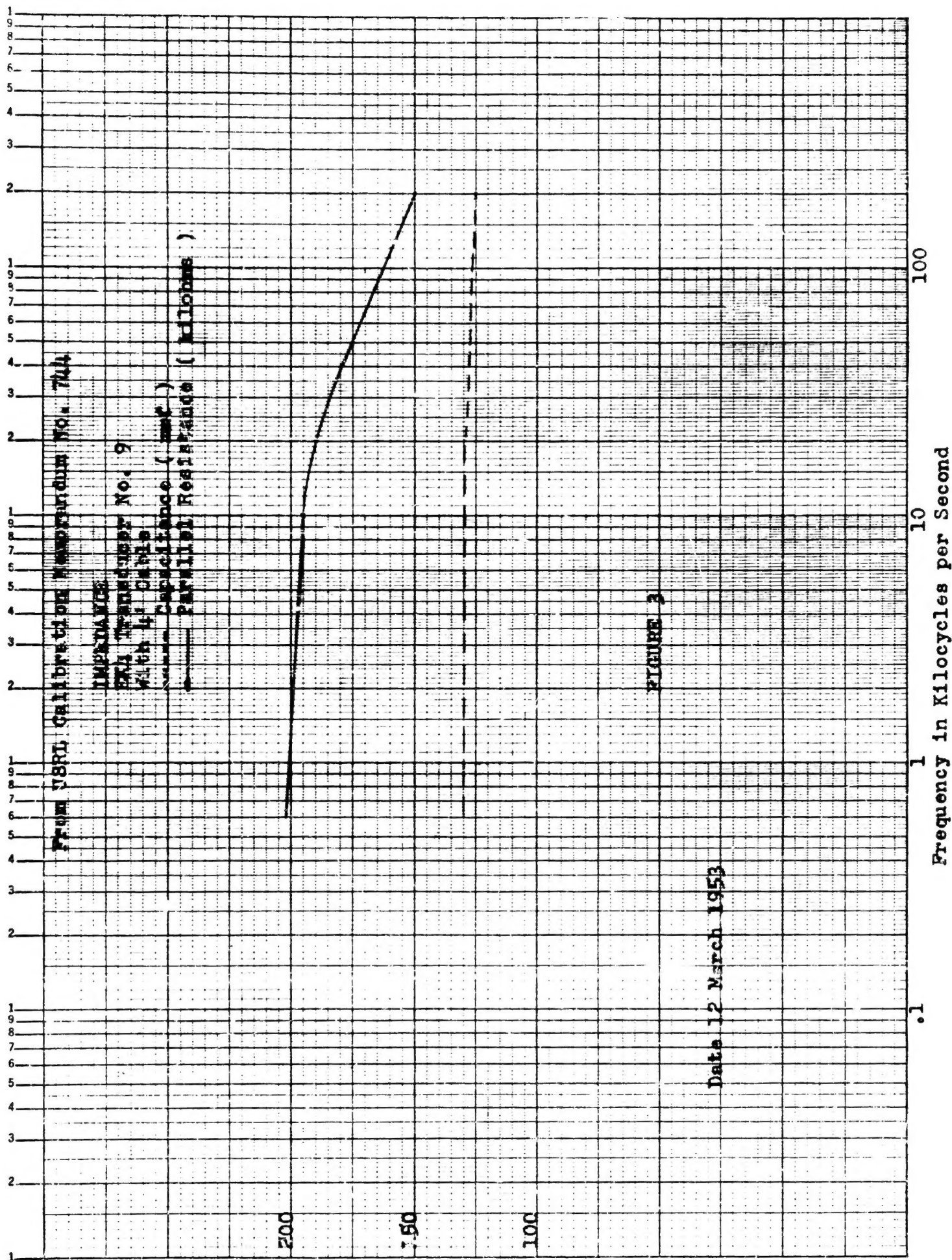
Figure 3 shows the results of impedance measurements made by the U.S.R.L. on the same transducer. (The change in











resistance from 10 to 100KC was not noted in impedance measurements made by The Beta Corporation. To a certain extent, however, these measurements depend on the current level because of polarization effects.)

The calculated noise voltage per cycle is -145DB re 1 volt flat to a 3DB break point at about 6,500 cps, the noise falling at 6DB per octave at higher frequencies. This corresponds to an equivalent noise pressure in one cycle of -42DB re 1 dyne per centimeter squared at frequencies below 2KC. The above figures are taken from U.S.R.L. Calibration Memo. No. 744.

#### 4. THE LF-1 HYDROPHONE

The LF-1 Hydrophone was developed as a pressure sensitive hydrophone with flat response to 0.1 cps. (A modified unit was assembled for a special project with response flat to 0.01 cps.) It was designed to include the decades from 10 cps to 0.1 cps in a design as such low frequencies are very difficult to cover with piezoelectric, ferroelectric, or electromagnetic devices.

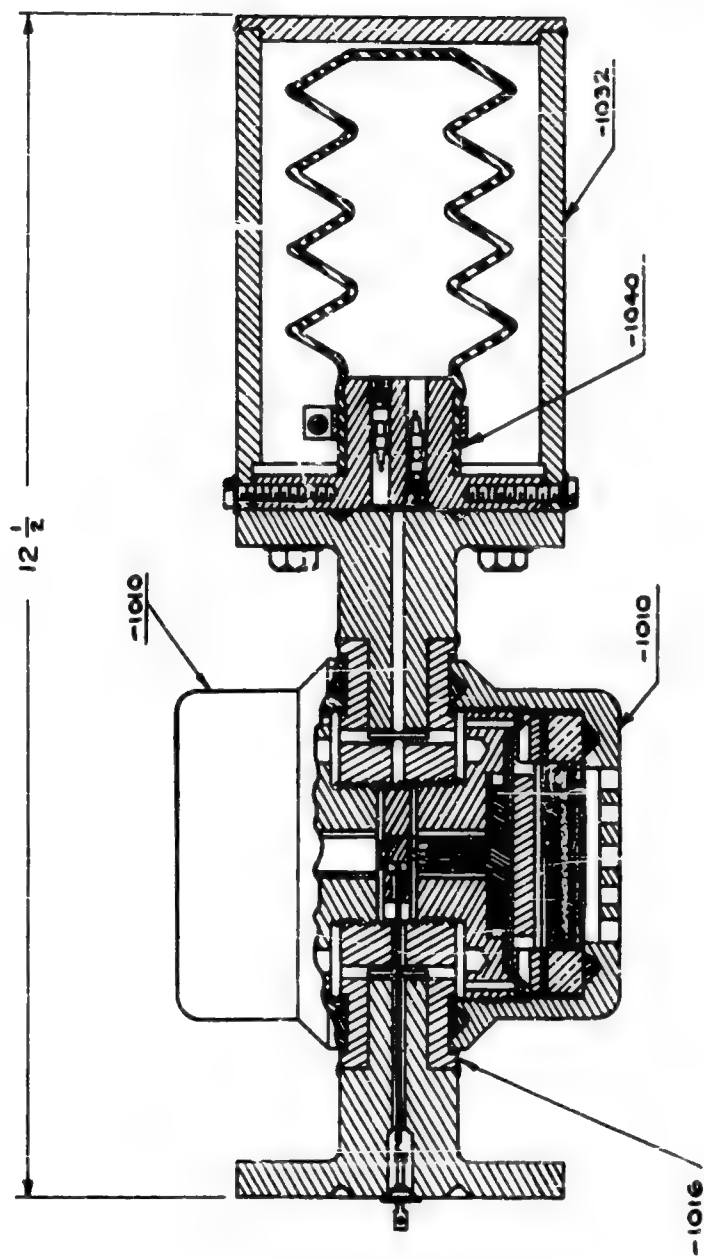
The LF-1 Hydrophone is shown in section in Figure 4 and as a photograph in Figure 5. Two 2" O.D. by 0.125" porous discs are employed in parallel connected cells in this hydrophone. Inertial effects are somewhat reduced in this opposed arrangement. The diaphragms are a heavy aluminum foil bonded to a Teflon impregnated glass cloth base, the latter being exposed to the outside. The equalizer is simply a neoprene air filled bellows exposed to the sea and connected through low pressure relief valves to the air passages in the hydrophone leading to the rear of the cells. As the hydrophone is lowered the pressure across the cells is equalized but their rears are acoustically isolated by the valves at any given depth. The LF-1 is designed to operate at up to 100 feet with the bellows illustrated. Two hundred feet of Spiral 4 cable were supplied with each LF-1 Hydrophone with one end sealed into the hydrophone cable gland.

An LF-1 Hydrophone, Serial No. 1, was shipped to the Underwater Sound Reference Laboratory on August 4, 1953 at the request of ONR. Some of the results of their tests as they were reported to ONR at intervals are as follows:

Measurements to check the sensitivity of the hydrophone as a function of pressure (0-50 psig) at 100 cps were made on the following dates and reported by letter, each of which stated that:

"No significant change from earlier results was detected."

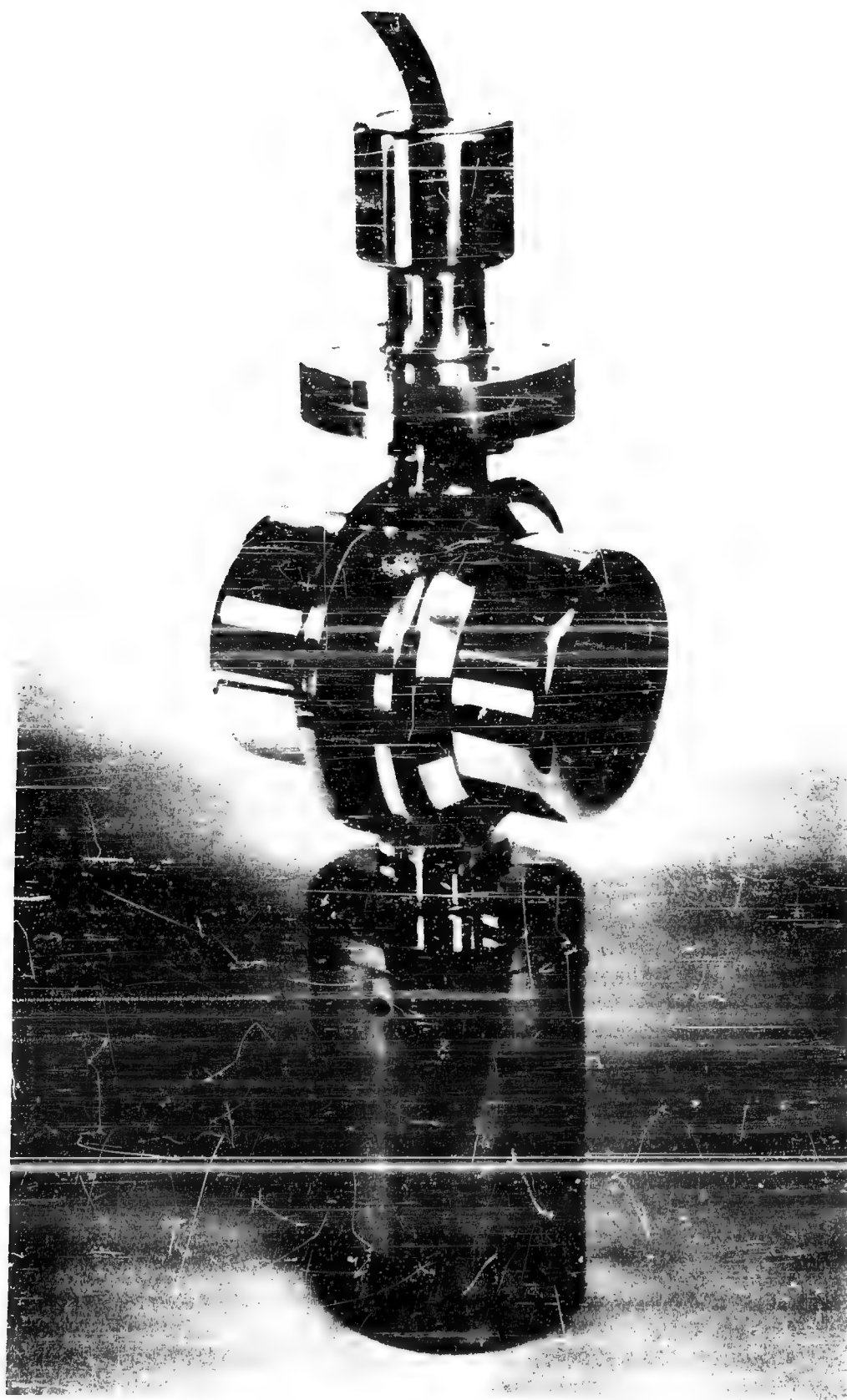
23 September 1953  
20 October 1953



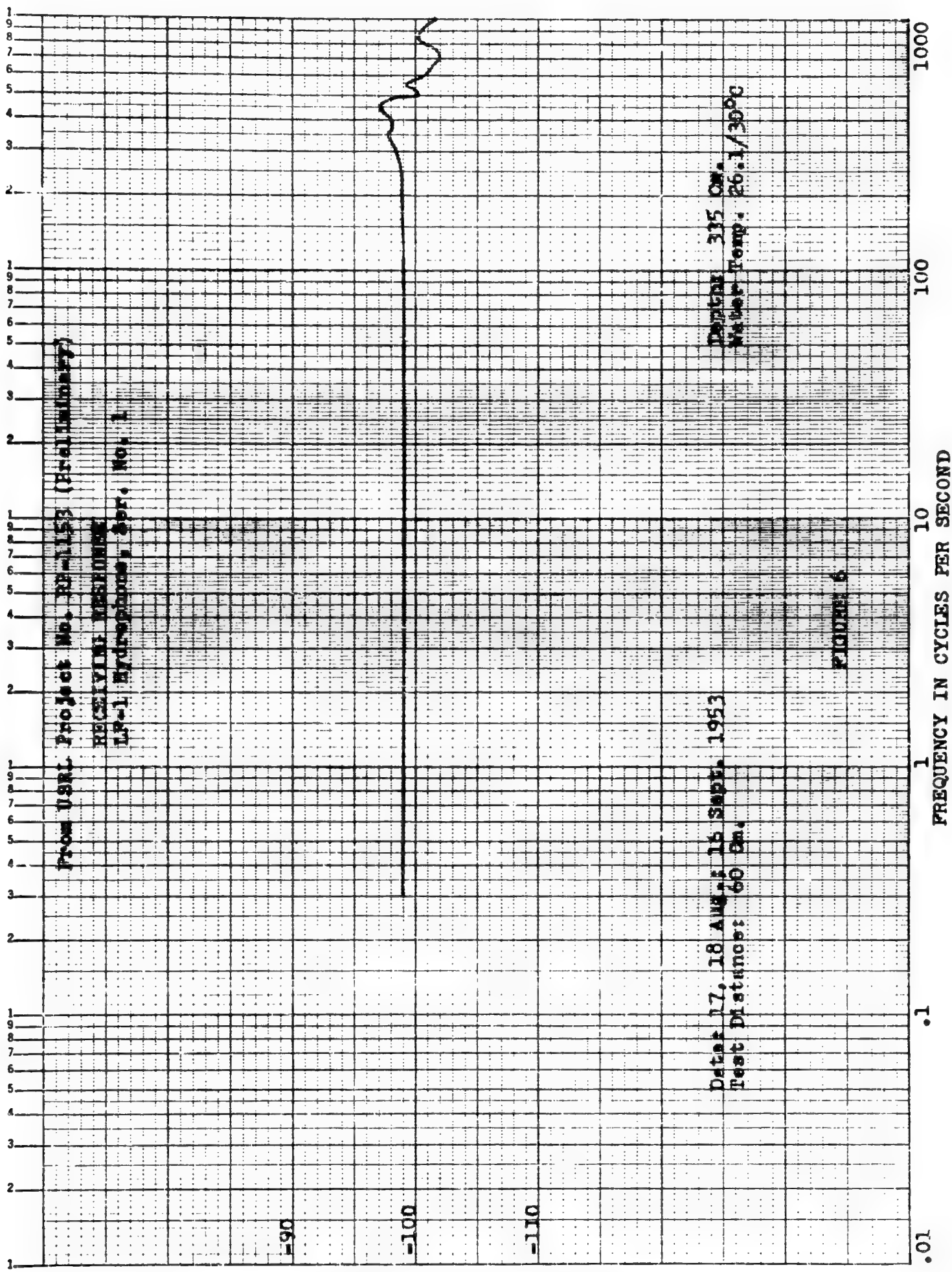
**FIGURE 4**

[illegible]





LF-1 HYDROPHONE  
FIGURE 5



SENSITIVITY IN DB RE 1 VOLT PER MICROBAR



24 November 1953  
18 December 1953  
20 January 1954

(References 2, 3, 4, 5, 6.)

Reference 2 reported a conductance of 60 micromhos (16,700 ohms) in the range from 200 cps to 1000 c.p.s. "The measured parallel capacitance over the same frequency range is 11,490 micromicrofarads". Their preliminary data showed a noise voltage of -156DB in one cycle re 1 volt at 200 cps and a sensitivity of -99DB. The frequency response curve also given as preliminary data is shown in Figure 6. Reference 7 confirmed that the response was found to be flat over a range from 0.3 to 2 cps hence these points are included in Figure 6.

#### 5. THE LF-2 HYDROPHONE

The LF-2 Hydrophone is shown in section in Figure 7 and as a photograph in Figure 8. In developing this hydrophone it was desired to evolve an instrument capable, if necessary, of working to depths of up to 20,000 feet without sub-surface preamplifiers and with the best low frequency response found reasonable to achieve in a pilot design.

The section drawing in Figure 7 is almost self-explanatory. The transducer unit, which is generally similar to the "Type 4", is located between the two oil filled cavities, only one of which is exposed acoustically to the sea. The entire assembly is filled with a viscous compressible oil permitting hydrostatic equalization throughout. A by-pass capillary with a very high flow resistance allows for changes in the specific volume of the trapped oil and thus prevents damage of the diaphragms while acoustically isolating the two chambers. The S.I.E. Type RI-1714 transformer matches the 150K transducer impedance to the cable at 500 ohms. A surface transformer, S.I.E. Type TI-1514, matches the 500 ohm line to the first grid at a 450K level. No grid resistor is used. The transformers provide response flat from a 2DB drop at 5 cps to well over 1000 cps or less depending on the cable length. The method of employing a compliant volume of oil as an equalizer for great pressures is treated quantitatively in Appendix A.

An LF-2 Hydrophone, Ser. 1, was sent to the Lamont Geological Observatory, Palisades, New York, for hydrostatic tests to 10,000 psi. The results of the tests are given in Reference 10 and are summarized briefly here. The hydrophone was sealed in a water filled tank and connected through a matching transformer to a Brush recorder. A standard signal was produced by dropping a lead weight 4" to the floor and the record thus





LF-2 HYDROPHONE  
FIGURE 8

-94  
 -100  
 -106  
 -112  
 -118  
 -124



produced was observed as the hydrostatic pressure was changed. Pressure was then slowly varied in cycles of 0-100-0 psi and 0-100-1000-100-0 psi with no change in sensitivity being noted. In a third run pressure was raised to 10,000 psi with stops at 100,1000, and 7000 psi. At 7000 and 10,000 psi the sensitivity decreased and the hum pickup increased. This effect continued down to the low pressures. The instrument was returned and disassembled in our laboratory. It was found that the large O-ring seal allowed some water to leak in which caused the difficulty. A modification of the seal is being made on several LF-2 Hydrophones to be tested at great depths.

Tests on the transformer and the transducer which had been subjected to 10,000 psi pressured showed no effects of the high pressure. Thus, there is evidence that transducers of the construction shown will function satisfactorily to 10,000 psi pressures.

An LF-2 Hydrophone, Serial No. 6, was tested by the Underwater Sound Reference Laboratory and the results were reported in Reference 8. Measurements were made at the 450,000 ohm grid winding of the matching surface transformer. The frequency response data is given in Figure 9. The cut-off at around 20 cps rather than at 5-10 cps indicates that in this unit some leakage of the oil exists past the transducer other than through the capillary causing a decrease in the time constant. This point will require more verification as further tests are conducted on the hydrophones. U.S.R.L. reported that this hydrophone withstood repeated exposures to 1000 psig pressures without changing its sensitivity but it was not tested by them at 1000 psig.

## 6. THE S-2 HYDROPHONE

The S-2 Hydrophone is shown in Section in Figure 10 and as a photograph in Figure 11. This experimental hydrophone was designed to investigate the feasibility of mechanical matching or "pressure multiplication" at shallow depths with an electrokinetic cell. The ratio of the effective area of the conical receiving diaphragm to that of the piston is approximately 8:1 giving a gain of about 18DB. Excluding the cell and the compressible air filled equalizer tube, the entire assembly is filled with silicone oil in pressure equilibrium. The combined action of the two areas and equalizer are discussed in Appendix A. The cell resistance is softer by a factor of 64 as reflected to the receiving diaphragm making the equalization problem more difficult in proportion to the realized power gain. An air filled internal equalizer of this type becomes stiffer with the square of the absolute pressure which is a distinct disadvantage as it causes low frequency response to fall off rapidly with depth. A linear spring equalizer would be a better solution where both

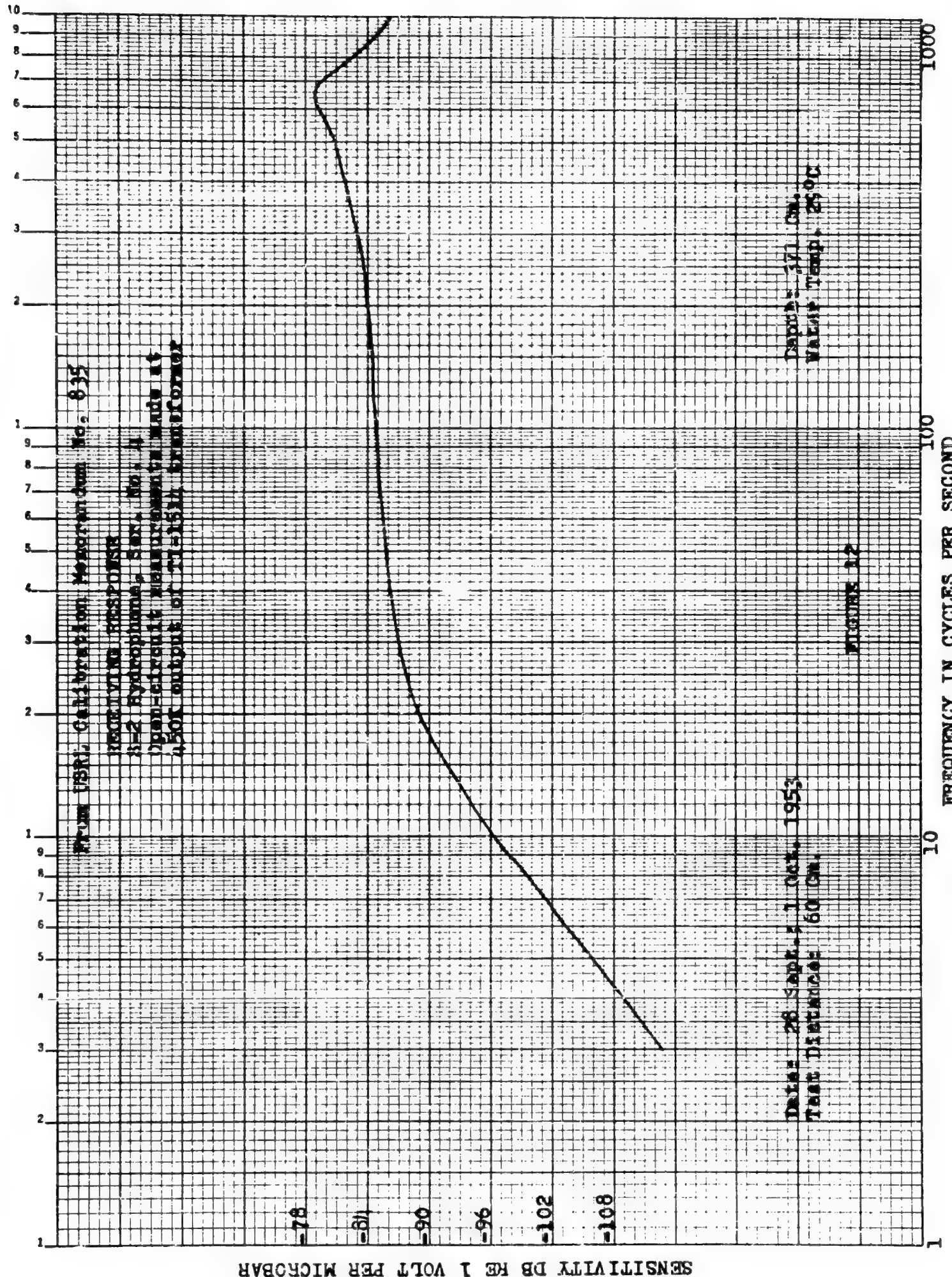


WG NO.  
300-3-1001





S-2 HYDROPHONE  
FIGURE 11



From USRI Calibration Memorandum No. 835  
RECEIVING RESPONSE  
H-2 Hydrophone, Ser. No. 1  
Open-circuit measurement made at  
450K output of T1-251H transformer

Date: 26 Sept. 1 Oct. 1953  
Test Distance: 60 cm.

Depth: 171 cm.  
Water Temp. 25°C

FIGURE 12

SENSITIVITY DB RE 1 VOLT PER MICROBAR

FREQUENCY IN CYCLES PER SECOND

an internal equalizer and mechanical matching are combined in the same design.

A Type S-2 Hydrophone, Serial No. 4, was tested by the Underwater Sound Reference Laboratory and the results are reported in Reference 9. Data obtained in these tests are shown in Figure 12. The frequency response characteristics of the "multiplier" are evident.

The use of mechanical matching devices is discussed further in Appendix A.

## 7. REFERENCES

1. U.S.R.L. Calibration Memorandum No. 744, Project No. IC-1008LNS, dated 15 April 1953.
2. U.S.R.L. ltr JMT/hs, RP-1153, T231-53 dated 23 September 1953.
3. U.S.R.L. ltr JMT/hs, RP-1153, Ser.T-266-53 dated 29 October 1953.
4. U.S.R.L. ltr JMT/am, RP-1153, Ser.T-296-53 dated 2 December 1953.
5. U.S.R.L. ltr JMT/am, RP-1153, Ser.T-008-54 dated 11 January 1954.
6. U.S.R.L. ltr JMT/am, RP-1153, Ser.T-028-54 dated 28 January 1954.
7. U.S.R.L. ltr JMT/am, RP-1153, RP-1209, Ser.T-014-54 dated 13 January 1954.
8. U.S.R.L. ltr JMT/am, RP-1209, Ser.T-010-54 dated 11 January 1954.
9. U.S.R.L. ltr JMT, RR-1154, Ser. T-261-53, dated 26 October 1953 and attached Calibration Memorandum 835 dated 21 October 1953.
10. Lamont Geological Observatory ltr RE: JRN;mpr dated Dec. 1, 1953.
11. Hardway, E. V., Instruments, Vol. 26, No. 8, p. 1186, August 1953.
12. The Beta Corporation, Technical Report No. 2 to ONR, Contract Nonr-617(00) dated December, 1952.
13. The Beta Corporation, Technical Report No. 3 to ONR, Contract Nonr-617(00) dated July, 1953.
14. U.S. Patents 2,615,940, 2,644,900, 2,644,901, 2,644,902, 2,661,430.

Note: Tables of references to prior work are given in 12 and 13 above.

## APPENDIX A

### DESIGN AND PERFORMANCE RELATIONSHIPS WITH ALIGNMENT CHARTS

#### (1) General:

Inasmuch as electrokinetic hydrophones are best adapted to the measurement and detection of sound pressures at low frequencies and at great depths, (without the use of preamplifiers) it is desirable to relate the various quantities governing their performance and size in such applications without reference to their particular configuration or design. It will be shown that for a given power sensitivity or equivalent noise pressure, and low frequency (3DB) break point, a limitation is immediately imposed on an equalization stiffness factor "JE". This and the operating depth may be related to the required minimum volume of a liquid or gas required if a compressible volume is to be the means chosen for equalization of the hydrostatic pressure.

In other words, it is possible to determine the basic design constants including minimum size for an electrokinetic hydrophone directly from the performance requirements without detailed consideration of the size or number of the transducing elements or porous plugs and without making any assumptions as to whether or not mechanical impedance matching means are to be used.

Relations are also given and charted to determine the minimum gas or oil equalization volumes required. Other means of equalization incorporating stiff diaphragms or linear springs may also be used; their volumetric stiffness being simply "JE".

High frequency response is determined by the particular design and construction of the hydrophone and other factors which will not be considered in this Appendix.

#### (2) Voltage Sensitivity, Impedance, Power Sensitivity, and Equivalent Noise Pressure:

The sensitivity of a hydrophone can best be expressed in terms of either its power sensitivity or its equivalent noise pressure in a one cycle bandwidth.

It is evident that when a transformer is used the voltage sensitivity or impedance can be changed at will within the limitations imposed by the transformer. This problem should,

therefore, be treated separately. Inasmuch, however, as flutter noise and other noise arising in the surface amplifier may impose a limitation on the minimum voltage sensitivity, and since data on hydrophones are often given in terms of voltage sensitivity, it is convenient to be able to simply relate these quantities on a chart. The following definitions will be used:

$S_V = (H/P)$ , the open circuit voltage sensitivity in volts per dyne/cm<sup>2</sup>.

$S_V = 20 \log (H/P)$  or voltage sensitivity in DB re 1 volt per dyne/cm<sup>2</sup>.

$R$  = Output impedance or resistance in ohms at the same level at which  $S_V$  is determined.

$S_P = 25,000 (H/P)^2 \times 1/R$ , the power sensitivity given in milliwatts for a 10 dyne per Cm<sup>2</sup>. sound pressure.

$S_P = 10 \log S_P$  or power sensitivity in DB re 1 milliwatt for 10 dynes per Cm<sup>2</sup>.

From the above definitions we obtain the well known relation:

$$S_P = S_V - 10 \log R + 44$$

This relation is included in the "P<sub>N</sub>" alignment charts.

## (2) Noise Pressure:

The thermal noise per cycle obtained from the formula for Johnson noise and arbitrarily defined at 25°C is:

$$e = 1.27 \times 10^{-10} \sqrt{R}$$

From the definition of power sensitivity we may obtain:

$$P = \frac{H}{\sqrt{S_P}} \frac{\sqrt{25,000}}{\sqrt{R}}$$

Substituting the noise voltage "e" for the signal voltage "H" we obtain the equivalent noise pressure in one cycle:

$$P'_N = \frac{\sqrt{25,000}}{\sqrt{S'_p}} \cdot 1.27 \times 10^{-10}$$

Expressing  $P'_N$  as a logarithm:

$$P_N = 10 \log S'_p - 154$$

$$P_N = -S_p - 154$$

From Equation (1):

$$P_N = -S_v + 10 \log R - 198 \quad (2)$$

The above relation is also included on the " $P_N$ " Chart. It should be recalled that it only applies at mid-band frequencies and at 25°C. In hydrophones, however, errors due to temperature are small because of the limited range involved.

The total noise pressure varies with the noise pressure per cycle times the square root of the bandwidth. Its allowable magnitude for a given set of conditions as to sea noise or signal pressure in combination with the frequency response limits define the performance requirements for a hydrophone.

### (3) The Equalization Factor " $J_E$ ", Noise Pressure, and Low Frequency Response:

Although no specific configurations need to be defined in deriving the final relations which follow, a simplified schematic arrangement showing a piston type "pressure multiplier" for mechanical impedance matching will assist in following the reasoning involved. Such an arrangement is shown in Figure 13.

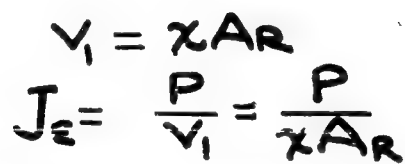
We first assume an ideal arrangement wherein all the energy absorbed from the sea is dissipated or converted in the porous plug of area " $A_p$ " and thickness " $t$ " by the flow of the polar liquid through the plug. In previous technical reports it was shown that only the resistive mechanical impedance of the plug need be considered.

In the derivation which follows only the sound pressures are considered, the equalized hydrostatic pressure being neglected.

It can be readily shown by derivation or from energy considerations in Figure 13 that:

$$P_1 \dot{V}_1 = P_2 \dot{V}_2$$





LET.	CHANGES	BY	PART NO.	NO. REQ.	DESCRIPTION
TOLERANCES		MATERIAL	NO. REQ.	TITLE <b>SCHEMATIC HYDROPHONE</b>	
DEC. _____					
FRACT. _____					
ANG. _____				<b>THE BETA CORPORATION RICHMOND, VA.</b>	
SCALE		SPEC.			
NEXT ASSEM.		FINISH		DRAWN _____ CHECK _____	DWG. NO.

Hence:

$$P_2 = P_1 \dot{V}_1 / \dot{V}_2$$

In the above  $P_1$  is the sound pressure acting on the receiving diaphragm of area  $A_R$ ,  $\dot{V}_1$  is the input volume rate or  $A_R \dot{x}$  where  $\dot{x}$  is the velocity of the diaphragm shaft and piston,  $P_2$  is the pressure acting on the porous plug and  $\dot{V}_2$  is the volumetric rate of flow through the porous plug of the polar liquid.

Since the piston and the receiving diaphragm move together:

$$\dot{V}_2 = \frac{A_P}{A_R} \dot{V}_1$$

where  $A_P$  is the piston area.

The pressure  $P_2$  is therefore greater than  $P_1$  by the area ratio:

$$P_2 = P_1 \frac{A_R}{A_P}$$

The electrical resistance of the porous plug is given in terms of  $K_o$ , the overall conductivity of the liquid filled plug as follows:

$$R = \frac{t}{K_o A_D}$$

If the disc sensitivity,  $(H/P)_D$  is increased by the area ratio to obtain the hydrophone power sensitivity we may write the power sensitivity using the above relations as:

$$S'_P = 25,000 \left( \frac{H}{P} \right)_D^2 \left( \frac{A_R}{A_P} \right)^2 \frac{K_o A_D}{t}$$

From this relation, it would appear that the power sensitivity might be increased indefinitely. It is evident, however, that the receiving diaphragm must have a neutral position and a restoring force to limit its travel under changes in pressure. In practice this force or pressure is determined by the equalizer necessary to prevent destruction of the transducing cell by the large hydrostatic pressure. It is convenient to

define an equalization factor  $J_E$  which is the ratio of the restoring pressure to a corresponding change in volume or  $A_R X$ .  $X$  is the corresponding displacement of the diaphragm.

Therefore:

$$J_E = \frac{P}{A_R X} = \frac{F_R}{A_R^2 X}$$

Where  $P_R$  is the restoring pressure and  $F_R$  is the equivalent restoring force on the piston.

The viscous damping force on the piston is given by:

$$F_D = \frac{\dot{V}_2 t A_P}{A_D k_v} = \frac{A_P^2 \dot{X} t}{A_D k_v}$$

where  $k_v$  is the flow conductance of the liquid filled plug.

The low frequency time constant is simply the ratio of the damping constant to the restoring constant:

$$\tau = (F_D / \dot{X}) (X / F_R) = \frac{A_P^2 t}{A_D k_v J_E A_R^2}$$

and the low frequency 3DB break point  $n_L$  below which the response drops 6DB per octave is given by:

$$n_L = \frac{1}{2\pi \tau} = \frac{A_D k_v J_E}{2\pi t} \left( \frac{A_R}{A_P} \right)^2$$

and

$$\left( \frac{A_R}{A_P} \right)^2 = \frac{2\pi n_L t}{A_D k_v J_E}$$

Substituting for the area ratio squared in the above relation for power sensitivity:

$$S_P' = 25,000 \frac{2\pi n_L}{J_E} \left[ \frac{K_o}{k_v} \left( \frac{H}{P} \right)_D^2 \right]$$

In Technical reports Nos. 1 and 2, it was shown that the sensitivity  $(H/P)$ , the electrical conductivity  $K_o$  and the flow conductivity  $k_v$  are all properties of a given liquid-solid combination and are independent of the size or thickness of the plug. The combination

$$\frac{K_o}{k_v} \left( \frac{H}{P} \right)_D^2$$

as a dimensionless parameter represents the transducer efficiency

or ratio of energy converted to that absorbed.

It may, therefore, be implied that for a given liquid-solid combination:

- (a) For a given requirement as to power sensitivity or noise pressure and low frequency response, the maximum allowable value of  $J_E$  is determined and the equalization requirements are established.
- (b) The above relationship is independent of the size or thickness of the porous plug, the number of plugs employed, or the type of impedance matching device employed, if any.
- (c) Mechanical impedance matching means permit a choice to be made as to whether to use a large plug area, a multiplicity of plugs, or a small plug with mechanical amplification. The fundamental equalization requirements are, however, unaffected.

The  $J_E$  Chart relations depend on the electrokinetic parameters ( $H/P$ ),  $k_v$ , and  $K_o$ . Each of these is subject to considerable variations but the variations tend to cancel the way that they are combined to represent the conversion efficiency of the liquid filled solid. Taking data from Figure 4-17 in Technical Report No. 3 at 25°C, for Grade #03 porcelain and acetonitrile:

$$K_o = 8 \times 10^{-7} \text{ mhos/Cm}$$

$$H/P = 4.9 \times 10^{-6} \text{ volts/dyne/Cm}^2.$$

$$k_v = 1.63 \times 10^{-8} \text{ Cm}^4/\text{dyne Sec.}$$

$$(\text{Efficiency} = 1.2\%)$$

Using the above values in the relation for power sensitivity derived in 3.0 we obtain:

$$S_p = -10 \log J_E + 10 \log n_L - 73.6$$

where  $J_E$  is in psi per  $\text{In}^3$ .

As previously derived:

$$P_N = -S_p - 154$$

And combining the above:

$$P_N = 10 \log J_E - 10 \log n_L - 80.4$$

The above relation is given charted on the enclosed  $J_E$  Chart.

(4) Equalization Volume:

Once the required  $J_E$  value has been determined, consideration must be given to the means of equalization. Such means may consist of a compressible oil volume, a compressible gas volume, a stiff diaphragm, a bellows backed by a linear spring, a compressed gas tank with regulating and escape valves or others. Some means of preventing the hydrostatic pressure from appearing across the electrokinetic cell diaphragms which contain the highly purified polar liquid is unavoidable if good low frequency response is to be obtained at appreciable depths. For operation at a given depth two requirements must be met.  $J_E$  must be equal to or less than the required value and the difference pressure across the cell must be small, generally less than 5 psi.

To minimize sealing problems and accomplish a low differential across the cell two means of equalization suggest themselves. One is to use a volume of gas behind the cell or receiving diaphragm which is compressed on descent to equalize the outside pressure and provide compliance. The other is to use a volume of oil. One method is the use of fixed and variable volumes where the fixed volume is in pressure contact with the rear diaphragm of the transducer, and a variable volume is exposed to the sea and acoustically isolated by a capillary or relief valves from the fixed volume and rear diaphragm. For a gas the variable volume will be generally larger than the fixed volume whereas the converse is true for liquids. In either case as the fluid is compressed the value for  $J_E$  will increase, hence the design must be based on the maximum operating depth.

It will be convenient to consider total volume at atmospheric pressure as this factor " $V_T$ " most directly affects the size.  $V_T$  is the sum of the fixed volume and the variable volume at atmospheric pressure.

(a.) Gas Volumes:

For a given fixed volume of gas  $V_F$  at pressure ( $P_A : P_H$ ) the value of  $J_E$  is given by:

$$J_E = \frac{\gamma_g (P_A + P_H)}{V_F}$$

where  $P_A$  is atmospheric pressure,  $P_H$  is the head pressure due to the depth in the water and  $\gamma_g$  is the specific heat ratio or 1.4 for air or nitrogen.

The minimum variable volume is that which will compress to zero at a pressure  $(P_A + P_H)$ , hence for constant temperature:

$$P_A(V_F + V_V) = (P_A + P_H)V_F$$

Since the total volume  $V_T$  is the sum of  $V_F$  plus  $V_V$  we may derive from the above:

$$V_T = \frac{\gamma_g}{J_E P_A} (P_A + P_H)^2$$

The above relation may be written as:

$$V_T = \frac{(P_H + 34)^2}{78.5 J_E}$$

where  $V_T$  is in cubic inches,  $P_H$  is in feet of water and  $J_E$  is in psi/In<sup>3</sup>. In logs:

$$10 \log V_T = 20 \log (P_H + 34) - 10 \log J_E - 10 \log 78.5 \quad (4)$$

The above relation is given in the  $V_T$  chart for gases enclosed. It is equally applicable where a single compressible volume of gas is used, i.e. as opposed to a fixed and a variable volume.

It is evident that because of the square relation the required volume increases extremely rapidly with depth. It may be demonstrated that beyond 4,000 feet a liquid equalizer is smaller whereas at shallow depths a gas equalizer requires less volume.

#### (b.) Liquid Volumes:

For liquids:

$$J_E = \frac{\gamma_L B_T}{V_F}$$

where  $\gamma_L B_T$  is the adiabatic bulk modulus of the liquid at the operating hydrostatic pressure. Where the density and sound velocity are known the following relation may be used:



$$\gamma_L B_T = \rho c^2$$

The fixed volume required is:

$$V_F = \frac{\gamma_L B_T}{J_E}$$

In computing the variable volume the relation below may be used:

$$V_V = \Delta V_F = \frac{P_H V_F}{B_T}$$

Actually the value of  $B_T$  chosen for use in the above equation should be based on the compression at the given depth. Since for liquids the variable volume is substantially smaller than the fixed volume, it may be approximated by using the isothermal bulk modulus at the depth in question. It is generally necessary to allow a safety factor in design for thermal expansion and contraction. Then:

$$V_T = V_F + V_V = \gamma_L B_T / J_E + P_H V_F / B_T$$

$$V_T = \gamma_L / J_E (B_T + P_H)$$

Unfortunately,  $B_T$  is a function of the depth of  $P_H$  and dependent on the particular liquid used. It has been found that silicone oils are not only inert and excellent insulators, but are also "soft" with low bulk moduli. Their low temperature - viscosity coefficients are also advantageous.

A chart has been prepared to enable  $V_T$  to be determined for 0.65 and 1000 centistoke silicone oils with oils of intermediate viscosities falling between the two. Since:

$$V_T J_E / J_E = V_T$$

and:

$$\log V_T = \log V_T J_E - \log J_E \quad (5)$$

the alignment chart for  $V_T$  for oils permits  $V_T$  to be determined. The value of  $J_E V_T$  is obtained from:

$$V_T J_E = \gamma_L (B_T + P_H) \quad (6)$$

which, for a given oil is a function of depth. The correction for depth for the two oils mentioned is given as a graph on one arm of the  $V_T$  alignment chart.

(5) Summary of Symbols, Definitions and Units Used on Alignment Charts:

$S_V$  = Open Circuit voltage sensitivity in DB re 1 volt per dyne/ $\text{Cm}^2$ .

$S_P$  = Power sensitivity in DB re 1 milliwatt at 10 dynes/ $\text{Cm}^2$ .

$P_N$  = Equivalent noise pressure per cycle in DB re 1 dyne/ $\text{Cm}^2$ .

$R$  = Output resistance in ohms.

$N_L$  = Low frequency break point, (where sensitivity drops 3DB) in c.p.s.

$J_E$  = Equalization stiffness factor in psi per  $\text{In}^3$  at receiving surface.

$P_H$  = Head or depth in feet of water.

$V_T$  = Total volume of equalization fluid at atmospheric pressure in cubic inches.

(6) Use of the Alignment Charts and Examples:

(a) The  $P_N$  Chart:

Assume for example that a requirement for the LF-2 hydrophone be that it provide a sensitivity of at least minus 100DB at a grid level open circuit impedance at 450K. The line drawn on the  $P_N$  chart shows the required power sensitivity to be -112DB and the noise pressure per cycle to be -42.5DB. (Actually the cell output in the LF-2 transformed to a 500 ohm line level and again to 450K in a surface transformer.)

(b) The  $J_E$  Chart:

Connect -42DB for  $P_N$  to 5 c.p.s. the low frequency limit of the LF-2 hydrophone at maximum depth or 20,000 feet. A value of 30,000 is obtained for  $J_E$ .

(c) The  $V_T$  Chart (Liquids):

Assuming the use of a compressible volume of 1000 ctsk oil in the LF-2, a value of 320,000 is obtained for  $J_E V_T$  at 20,000 feet which corresponds to a volume  $V_T$  of approximately 11 cubic inches. Actually a compressible volume of 15 cubic inches of 200 centistoke oil was used giving a safety factor to allow for the stiffness of the diaphragm of the cell which adds about 1 c.p.s. to the low frequency limit.

(d) The  $V_T$  Chart (Gases):

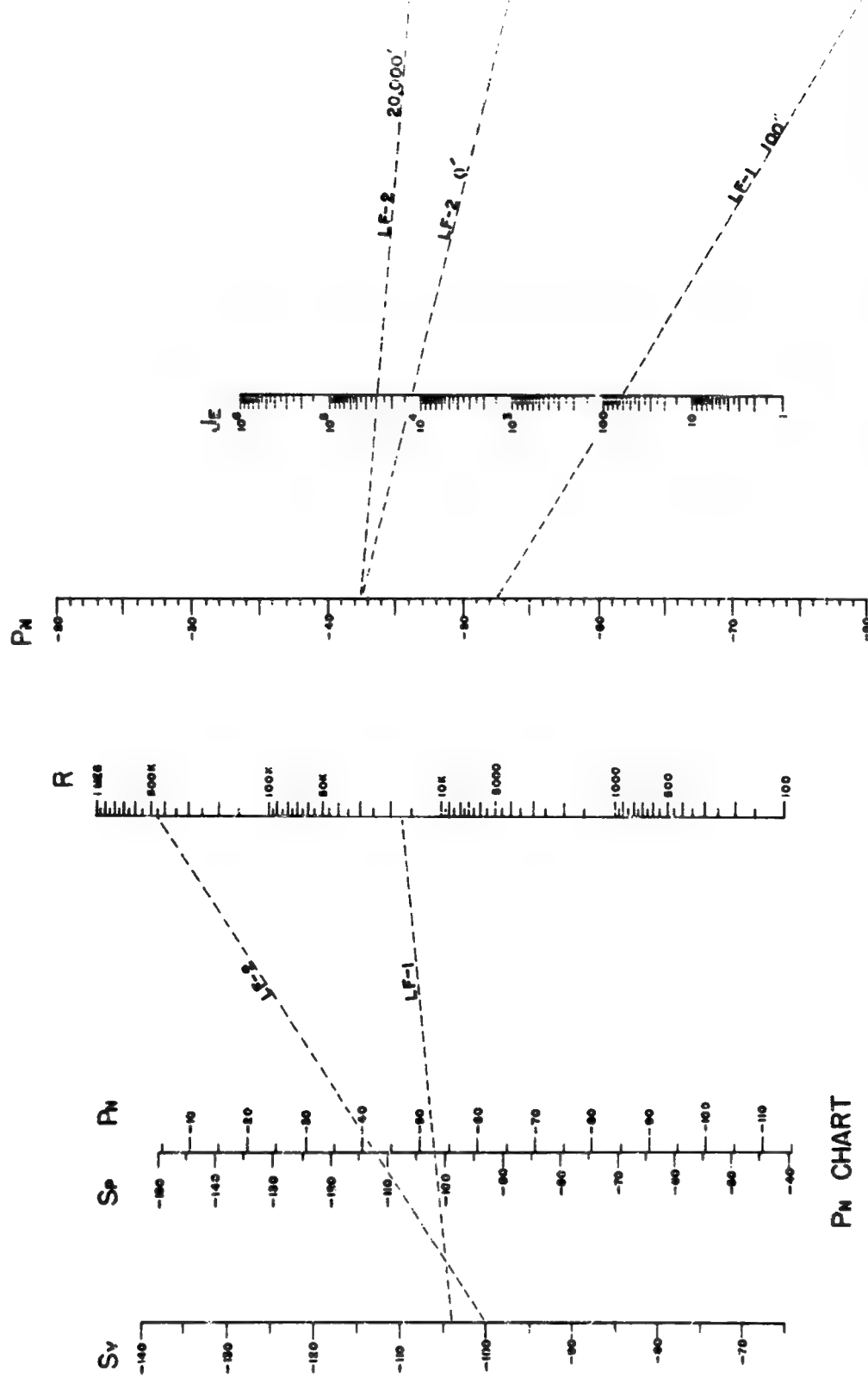
For the LF-1 assume that a  $P_N$  of -52.5DB was maximum and response down to 0.1 cps was required at 100 foot depths.  $J_E$  is therefore 60 or 500 times less than that for the LF-2.

Using the  $V_T$  chart for gases at a  $P_H$  of 100 feet and a  $J_E$  of 60, the value for  $V_T$  is found to be four (4) cubic inches. If the actual hydrophone a fixed volume of 1 cubic inch and a variable volume of  $4\frac{1}{2}$  cubic inches was used so that  $V_T = 5.5 \text{ In.}^3$ . This hydrophone used two parallel 2" OD x 1/8" plugs. The contribution of the diaphragms is small at 100' because of their larger diameter, hence the safety factor allowed for the volumes is ample.

(7) Conclusions:

At depths of less than 1000 feet gas volumes or other types of equalizers should be used. At depths of over 4000' only liquid filled equalizers should be considered. Whether or not mechanical or electrical impedance matching devices should be used will depend on other factors.

Equations have been derived which relate the performance parameters for a low frequency hydrophone to the principal factor controlling its size at various depths when an electrokinetic transducer is employed. The parameters are based on the use of Grade #03 porcelain and acetonitrile which form one of the most efficient material combinations known at the present time.



Je CHART

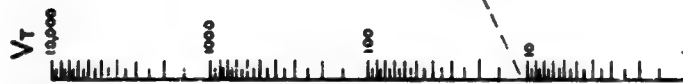
FIGURE 14

LET	CHANGED	BY	PART NO.	REV.	DATE
TOLERANCES			TITLE		
DEC.	MATERIAL		NO. REQ.		
FRAC.					
ANG.					
SCALE	SPEC.				
NEXT ASSEMB.		FINISH			

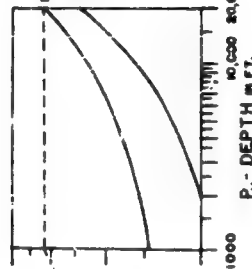
**Pn AND JE CHART**  
**THE BETA CORP.**  
**RICHMOND**  
**DRAWN V.L. 10-20-50**  
**CHECK**  
**DWG.**



**V<sub>T</sub> CHART**  
SILICONE OILS (CU. IN.)

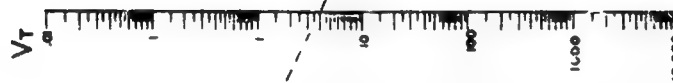


$V_T \times 10^{-6}$



P<sub>H</sub> - DEPTH IN FT.

P<sub>H</sub> IN FT. OF WATER



**V<sub>T</sub> CHART**  
AIR OR NITROGEN (CU. IN.)

FIGURE 15

LET.	CHANGES	BY	PART NO.	NO. REQ.	TITLE
TOLERANCES			NO. REQ.		
DEC.	MATERIAL		NO. REQ.		
FRAC.	SPEC.		NO. REQ.		
ANG.	FINISH		NO. REQ.		
SCALE	NEXT ASSEN.		NO. REQ.		

THE BETA CORP.  
RICHMOND, VA.  
DRAWN V-11-2753 DWG  
CHECK

## APPENDIX B

### SELF-NOISE AT LOW FREQUENCIES

The question often arises as to how the thermal self-noise in an electrokinetic hydrophone compares with that of a piezoelectric or ceramic hydrophone which has "no" parallel resistive component. The thermal noise in a piezoelectric or ceramic hydrophone is apt to be negligible at high frequencies but may be quite large at low frequencies. A shunt resistive component will always exist. (If it didn't the hydrophone would measure to zero frequency.) As its value increases the noise power increases, but this is more than offset by the increased attenuation due to the presence of the capacity across the noise source.

In Figure 16 the thermal noise equivalent circuits are given for both electrokinetic and piezoelectric or ferroelectric transducers at low frequencies.

In each circuit  $E_1$  is the voltage proportional to the applied pressure  $P$ , the sensitivity being  $S$ . In each circuit the noise voltage is represented by a voltage source  $E_n$  in series with the resistance. External shunt resistance in the electrokinetic circuit can be neglected or accounted for in the value of  $R_e$ , the output resistance of the transducer.

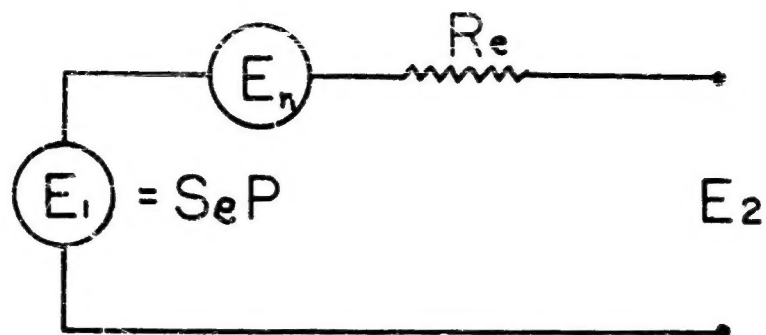
Since the noise voltage in one cycle  $e_{ne}$  for the electrokinetic element may be written simply as  $K\sqrt{Re}$  where  $K$  is the proper constant for a given temperature it is evident that the equivalent noise pressure for it will be:

$$P_{ne} = \frac{e_{ne}}{S_e} = \frac{K\sqrt{Re}}{S_e}$$

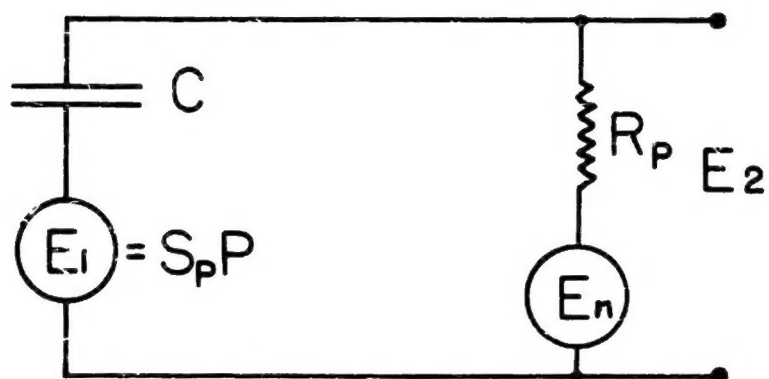
The piezoelectric circuit is somewhat more complicated. From the equivalent circuit we may write the voltage at component of  $E_2$  due to a pressure  $P$  as  $e_{np}$  where:

$$e_p = \frac{S_p R_p \omega C}{\sqrt{1 + (\omega C R_p)^2}} P$$

The noise voltage component of  $E_2$  in one cycle is the voltage  $e_{np}$  produced at the output terminals due to the thermal voltage  $E_n$ . This may be written as:



ELECTROKINETIC



PIEZOELECTRIC OR FERROELECTRIC

FIGURE 16

LET.	CHANGES	BY	PART NO.	No. REQ.	DESCRIPTION
TOLERANCES		MATERIAL	No. REQ.	TITLE EQUIVALENT CIRCUITS FOR NOISE COMPARISON  THE BETA CORPORATION RICHMOND, VA.	
DEC. _____					
FRACT. _____					
ANG. _____					
SCALE		SPEC.			
NEXT ASSEM.		FINISH		DRAWN _____ CHECK _____	DWG. NO.



$$e_{np} = \frac{K \sqrt{R_p}}{\sqrt{1 + (\omega C R_p)^2}}$$

From the above we obtain the equivalent noise pressure in one cycle appearing across the output terminals:

$$P_{np} = \frac{K \sqrt{R_p}}{S_p R_p \omega C}$$

For comparison we ratio the noise pressure of the piezoelectric to the electrokinetic:

$$\frac{P_{np}}{P_{ne}} = \frac{S_e}{S_p} \cdot \sqrt{\frac{R_p}{R_e}} \cdot \frac{1}{R_p \omega C}$$

Now  $R_p C$  is the reciprocal of the low cut-off frequency of the piezoelectric device, hence the ratio may be rewritten as:

$$\frac{P_{np}}{P_{ne}} = \frac{S_e}{S_p} \cdot \sqrt{\frac{R_p}{R_e}} \cdot \frac{n_L}{n}$$

where  $n$  is the frequency in c.p.s. and  $n_L$  is the measured or known cut-off frequency of the crystal. To eliminate the value  $R_p$ , frequently unknown, we may write:

$$\frac{P_{np}}{P_{ne}} = \frac{S_e}{S_p} \cdot \frac{\sqrt{n_L}}{n} \cdot \frac{1}{\sqrt{R_e C}}$$

A numerical example will illustrate the utility of the above equation. Assume it is desired to compare the equivalent noise pressure per cycle of an electrokinetic hydrophone such as the LF-1 with an impedance of 17K and a sensitivity of, say, -103DB with a piezoelectric or ceramic hydrophone which has a capacity of .005 mfd, a sensitivity of -93DB and which cuts off at 10 c.p.s. The relation then becomes:

$$\frac{P_{np}}{P_{ne}} \approx \frac{100}{n}$$

In this typical example the equivalent noise pressures per cycle are equal at 100 cps. It is, however, worse by a factor of ten for the piezoelectric device at 10 cps in spite of its 10DB greater sensitivity. It is evident that the importance of the comparison depends on the anticipated level of sea noise.

## APPENDIX C

### TRANSFORMER NOTES

All transformers used on this project were procured from the Southwestern Industrial Electronics Co., P. O. Box 13058, Houston 19, Texas. Data on certain of these transformers are given below:

(a.) TI-1514 Primary 500/125 CT, secondary 450,000 CT. Primary inductance at 10 MV is 9.7 henries. Primary D. C. resistance 60 ohms. Secondary D. C. resistance; 19,000 ohms. These transformers are flat from 5 to 1000 cps. They were used as surface input transformers for the LF-2 and S-2 hydrophones. They employ double Mu-metal and electrostatic shields.

(b.) RI-1714 is the epoxy impregnated equivalent of the RI-1201. Primary 500/125 C.T. used to match cable. Secondary 157,000 C.T. used to match transducer. Primary inductance at 10 MV is 8 henries. Primary D. C. resistance 100 ohms. Secondary D. C. resistance 9,500 ohms. Single Mu-metal and electrostatic shields. Tested in the LF-2 Hydrophone at pressures up to 10,000 psi. Response 5-1000 cps. (The RI-1201 was used in the S-2 Hydrophone.)

(c.) KO-1429 output transformer. Impedances 20,000 CT/5000 to 1000 CT/250 ohms. Turns ratio 4.5:1. Primary inductance 32,000 henries. Response 2 DB from 0.1 to 7,500 cps. Hum bucking coil construction with two Mu-metal and one copper nested shields. Size  $3\frac{1}{4} \times 3 \times 3\frac{1}{2}$ ". Wt.  $3\frac{1}{2}$  lbs. These transformers were not actually tested in service with the LF-1 Hydrophones but should be perfectly suitable for underwater and surface use to match a long low impedance cable at frequencies down to 0.1 cps.

(d.) No. 1546 Experimental transformers. The RI-1714 and TI-1514 transformer combination matches a 150K transducer to a 500 ohm line and in turn to a 450K level at the grid winding. This is a good combination except that the cut-off frequency is at 5 cps. The overall voltage gain is about 4.6DB. The problem of maintaining the same gain in a similar combination to work to 1 cps was not solved. The self-capacity of a high impedance winding suitable for 1 cps operation severely limits the high frequency response.

The 1546 transformers were made with dual primary and secondary windings to obtain different winding combinations.

Tests were made using a 150K source and a cathode follower grid circuit with two 1546 transformers matching to a network representing a 20,000 foot cable. The dummy cable consisted of two series 100 ohms resistors with a 1 mfd capacitor shunting their junction to the other wire and forming a "T". With this arrangement it was found possible to achieve unity gain from 4DB points at 1 cps and 500 cps. With another connection and a 6DB loss in gain the response was down 2DB at 1 cps. and 3DB at 500 cps. No loss occurred at 500 cps, when the transformers were directly connected ie without the dummy cable. The arrangement with unity gain employed a 24.6/1 and 1/24.6 turns ratios. In conclusion it was demonstrated that unity gain could be obtained with a 20,000 foot cable from a 150K transducer to a 150K grid level between 4DB points at 1 cps and 500 cps.